# Sample Paper-04
## Mathematics
## Class – XII

**Time allowed: 3 hours** **Maximum Marks: 100**

**General Instructions:**

(i) All questions are compulsory.
(ii) This question paper contains 29 questions.
(iii) Question 1- 4 in Section A are very short-answer type questions carrying 1 mark each.
(iv) Question 5-12 in Section B are short-answer type questions carrying 2 marks each.
(v) Question 13-23 in Section C are long-answer-I type questions carrying 4 marks each.
(vi) Question 24-29 in Section D are long-answer-II type questions carrying 6 marks each.

## Section A

1. Find gof f(x) = |x|, g(x) = |5x + 1|
2. Find $\lambda$, if the vectors $\vec{a} = \hat{i} + 3\hat{j} + \hat{k}, \vec{b} = 2\hat{i} - \hat{j} - \hat{k}$ and $\vec{c} = \lambda + 3\hat{k}$ are coplanar.
3. Find the value of $\cos(\sec^{-1} x + \operatorname{cosec}^{-1} x)$
4. A be a non – singular square matrix of order 3 ×3. Then $|adj\ A|$ is equal to

## Section B

5. Find $\tan^{-1} \frac{x}{\sqrt{a^2 - x^2}}$.
6. Solve for x given that $\begin{bmatrix} 2 & -3 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ 4 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$
7. Find $\frac{dy}{dx}$ $if\ \sin^2 y + \cos xy = \pi$
8. Find $\frac{dy}{dx}$ when x = a($\theta$-sin$\theta$), y = a(1+cos$\theta$)
9. $\int \cos^3 x . e^{\log \sin x} dx$
10. Solve the diff eq.
 $$\frac{dy}{dx} + \sqrt{\frac{1-y^2}{1-x^2}} = 0$$
11. Show that the points $A\left(2\hat{i} - \hat{j} + \hat{k}\right), B\left(\hat{i} - 3\hat{j} - 5\hat{k}\right), C\left(3\hat{i} - 4\hat{j} - 4\hat{k}\right)$ are the vertices of right angled triangle.
12. If A and B are two independent events, then the probability of occurrence of at least one of A and B is given by 1 – P ( $A'$) P ( $B'$).

**Section C**

13. Obtain the inverse of the following matrix using elementary operations $A=\begin{bmatrix}0 & 1 & 2\\ 1 & 2 & 3\\ 3 & 1 & 1\end{bmatrix}$

14. Sand is pouring from a pipe at the rate of 12cm$^3$/s. the falling sand forms a cone on the ground in much a way that the height of the cone is always one – sixth of the radius of the here. How fast is the height of the sand cone increasing when the height in 4cm.

15. A balloon, which always remains spherical on inflation, is being inflated by pumping in 900cm$^3$/s. find the rate at which the radius of the balloon increase when the radius is 15cm.

16. Integrate $\int \frac{dx}{x(x^4-1)}$.

17. Evaluate $\sin^{-1}\left(\frac{5}{13}\right)+\cos^{-1}\left(\frac{3}{5}\right)$

18. Show that the points A,B and C with position vectors $\vec{a}=3i-4j-4k, \vec{b}=2i-j+k, \vec{c}=i-3j-5k$ form the vertices of a right angled triangle.

19. The probability of solving a specific problem independently by A and B are ½ and 1/3 respectively. If both try to solve the problem independently, find the probability that (a) problem is solved (b) exactly one of them solves the problem.

20. A doctor is to visit a patient. From the past experience, it is known that the probabilities that he will come by train, bus, scooter or by other mean of transport are respectively $\frac{3}{10}, \frac{1}{5}, \frac{1}{10}$ and $\frac{2}{5}$. The probabilities that he will be late are $\frac{1}{4}, \frac{1}{3}$, and $\frac{1}{12}$ if he comes by train, bus and scooter respectively, but he comes by other means of transport, that he will not the late. When he arrives he is late. What is the probability that he comes by train.

21. Find all points of discontinuity of the function f where f is defined by:

$$f(x)=\begin{cases} x^3-x+1, x\le -3\\ -2x, -3<x<3\\ 3x+2, x\ge 3\end{cases}$$

22. Solve the differential equation $x\frac{dy}{dx}-y+x\operatorname{cosec}\left(\frac{y}{x}\right)=0,\ y(1)=0$

23. Find the vector equation of the plane passing through the intersection of the planes $\vec{r}.(2i+2j-3k)=7, \vec{r}.(2i+5j+3k)=9$ and the point (2, 1, 3)

**Section D**

24. Consider $f:\mathrm{R}_+\to[-5,\infty]$ given by $f(x)=9x^2+6x-5$. Show that $f$ is invertible with $f^{-1}(y)=\left(\frac{\left(\sqrt{y+6}\right)-1}{3}\right)$.

25. Using integration find the area of the region $\{(x,y): x^2+y^2\leqslant 2ax, y^2\geqslant ax, x, y\geqslant 0\}$.

26. Find the distance of the point (-1,-5,-10) from the point of intersection of the line $\vec{r} = (2\hat{i} - \hat{j} + 2\hat{k}) + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})$ and the plane $\vec{r}.(\hat{i} - \hat{j} + \hat{k}) = 5$
27. A factory can hire two tailors A and B in order to stitch pants and shirts. Tailor A can stitch 6 shirts and 4 pants in a day. Tailor B can stitch 10 shirts and 4 pants in a day. Tailor A charges 15 per day and tailor B charges 20 per day. The factory has to produce minimum 60 shirts and 32 pants. State as a linear programming problem and minimize the labour cost.
28. Solve the following system of equations using matrix method

$$\frac{3}{x} - \frac{2}{y} + \frac{3}{z} = 8$$

$$\frac{2}{x} + \frac{1}{y} - \frac{1}{z} = 1$$

$$\frac{4}{x} - \frac{3}{y} + \frac{3}{z} = 4$$

29. $\int_0^{\pi} \frac{x\,dx}{4\cos^2 x + 9\sin^2 x}$